Opportunistic Spectrum Sharing in Cognitive MIMO Wireless Networks

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***Abstract*—Cognitive radio has been recently proposed as a promising technology to improve the spectrum utilization. In this paper, we consider the spectrum sharing between a large number of cognitive radio users and a licensed user in order to enhance the spectrum efficiency. With the deployment of M antennas at the cognitive base station, an opportunistic spectrum sharing approach is proposed to maximize the downlink throughput of the cognitive radio system and limit the interference to the primary user. In the proposed approach, cognitive users whose channels are nearly orthogonal to the primary user channel are pre-selected so as to minimize the interference to the primary user. Then, M best cognitive users, whose channels are mutually near orthogonal to each other, are scheduled from the pre-selected cognitive users. A lower bound of the proposed cognitive system capacity is derived. It is then shown that opportunistic spectrum sharing approach can be extended to the multiple-input/multiple-output (MIMO) case, where a receive antenna selection is utilized in order to further reduce the computational and feedback complexity. Simulation results show that our proposed approach is able to achieve a high sum-rate throughput, with affordable complexity, when considering either single or multiple antennas at the cognitive mobile terminals.**

***Index Terms*—Cognitive radio, broadcast channels, spectrum sharing, multi-user scheduling, multiple-input/multiple-output.**

I. INTRODUCTION

THE explosive growth in wireless services over the past several years illustrates the huge and growing demand of the business community, consumers and the government for wireless communications. With this growth, the spectrum is becoming more and more congested. Even though the Federal Communications Commission (FCC) has expanded some spectral bands, these frequency bands are exclusively assigned to specific users or service providers. Such expansion does not necessarily guarantee that the bands are being used most efficiently all the time. Recent survey has in fact proved that most of the radio frequency spectrum is vastly under-utilized [1], [2]. For example, cellular network bands are overloaded in most parts of the world but amateur radio or paging frequencies are not. Moreover, those rarely used frequency bands assigned to specific services cannot be accessed by unlicensed users, even if the transmission of the unlicensed users does not introduce any interference to the licensed service. To deal with the conflicts between spectrum congestion and spectrum under-utilization, cognitive radio (CR) has been recently proposed as a smart and agile technology which allows non-legitimate users to utilize licensed bands opportunistically [3], [4]. By detecting particular spectrum holes and jumping into them rapidly, the CR can improve the spectrum utilization significantly. To guarantee a high spectrum efficiency while avoiding the interference to the licensed users, the CR should be able to adapt spectrum conditions flexibly. Hence, some important abilities should be provided by the CR which include spectrum sensing, dynamic frequency selection and transmit power control [5]. However, the interference caused by sharing the same radio channel becomes an obstacle that limits the whole system performance, such as the system throughput. Thus, when the cognitive user is sharing the spectrum with the primary user, the aim behind the system should be to maximize the throughput of the cognitive network without affecting the performance of the primary user.

Multiple-input/muliple-output (MIMO) systems have a great potential to enhance the throughput in the framework of wireless cellular networks [6], [7]. In fact, when using M transmit antennas at the base station and N receive antennas at the mobile user, the capacity of a MIMO single user is equal to $\min\{M, N\}$ times the capacity of a single-input/single-output (SISO) system [6],[7]. Multiple antennas can be applied to achieve many desirable goals for wireless communications, such as capacity increase without bandwidth expansion, transmission reliability enhancement via space-time coding, and co-channel interference suppression for multi-user transmission. By using multiple antennas in CR, one can allocate transmit dimensions in space and hence can obtain much design benefits to the MIMO cognitive network. In particular, we can obtain high spatial multiplexing gain by sending independent information streams over any transmit-receive antenna pair simultaneously to increase the system throughput of the cognitive radio system [8]. Moreover, multi-user interference can be suppressed by applying transmit beamforming [9]. Multiple antennas can be usually deployed at the base station, but they cannot be used easily at the mobile terminals due to the size and cost constraints. This may limit the capacity of the system when a limited number of antennas at the receivers is considered. The problem can be addressed by serving multiple users with single antennas simultaneously, and in this case, the system can be viewed as a virtual MIMO system.

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In a CR network, spectrum sharing can also be considered to further improve spectrum utilization efficiency. However, the primary user will always have a higher priority compared to the secondary users in utilizing the spectrum resources. Hence, one fundamental challenge of spectrum sharing is to ensure the quality-of-service (QoS) of the primary user, by keeping the interference caused to it limited. Therefore, it is crucial in the design of CR systems to take into consideration two conflicting objectives, namely, maximizing the throughput of the cognitive system and minimizing the interference at the primary receiver. In [10], [11] authors designed a capacity-achieving transmit spatial spectrum for a single secondary link in a CR network under both its own transmit-power constraint and interference-power constraint at the primary receivers. The proposed problem was formulated as a convex optimization problem. In [12], the problem of joint power control and beamforming in the downlink of the CR network was studied for a limited number of users.

In this paper, we consider a CR system where a large number of secondary users are operating in the same frequency band as the primary user. This will necessitate a scheduling of a few of them for transmission, and for this purpose, we develop an efficient opportunistic spectrum sharing approach. In particular, to guarantee the coexistence of the secondary users and the primary user on the same frequency band, we propose a low-complexity user scheduling algorithm, which we refer to as $\{\delta_p, \delta_c\}$-orthogonal user selection. By applying this algorithm along with zero-forcing beamforming (ZFB) and optimum power allocation, the proposed approach is able to achieve high system throughput and significant interference suppression. Besides, we shall derive a lower bound of the capacity of the cognitive radio system after scheduling M out of K cognitive users. Based on the lower bound, we are able to determine the optimal range of the parameter δ_c in order to improve the system performance.

Our proposed approach is also extended to the case of MIMO, where the users are using multiple antennas. In this case, each user firstly selects the most favorable receive antenna using a receive antenna selection algorithm. Secondly, a set of M users is determined through the $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm. It is shown that a significant reduction of the complexity of search during the scheduling process and required feedback is obtained when the receive antennas are selected, at the expense of a little loss in throughput.

The rest of this paper is organized as follows. In Section II, the system model and the problem formulation are introduced. In Section III, an opportunistic spectrum sharing approach is proposed. In Section IV, the system performance of the proposed algorithm is presented. In particular, multi-user diversity gain and the capacity analysis of the cognitive system are studied. In Section V, the MIMO case is considered, and hence a receive antenna selection is further proposed. In section VI, simulation results are presented followed by the complexity study of the proposed algorithm. Finally, conclusions are drawn in Section VII.

Throughout the paper, we use uppercase boldface letters for matrices and lowercase boldface for vectors. The Euclidean norm of a vector is denoted by $||\cdot||$. $(\cdot)^{\mathcal{T}}$, $(\cdot)^{\mathcal{H}}$, and $(\cdot)^{\dagger}$ stand for the transpose, the conjugate transpose and the pseudo-inverse, respectively. $\mathbb{C}^{x\times y}$ denotes the space of $(x \times y)$ matrix with complex entries. The distribution of a circularly-symmetric-complex-Gaussian vector with the mean vector x and the covariance matrix Σ is denoted by $\mathcal{CN}(x, \Sigma)$, and $\sim$ means "distributed as". $\mathbb{E}(\cdot)$ represents the expectation operator, and $|\mathcal{A}|$ denotes the cardinality of the set $\mathcal{A}$. $\lfloor\cdot\rfloor$ denotes the floor operation.

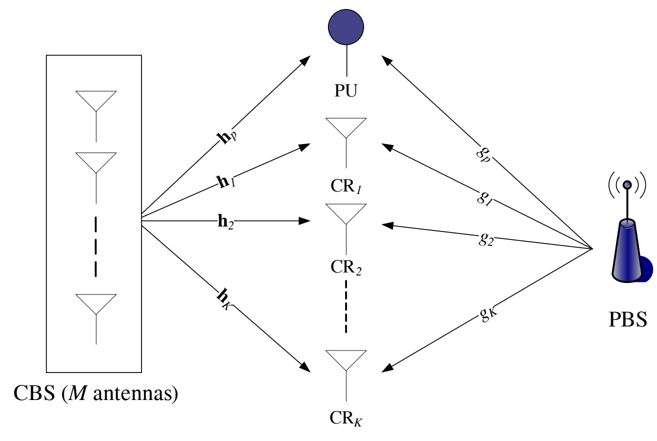

Fig. 1. System model. CR and PU denote the cognitive radio and the primary user, respectively.

II. System Model and Problem Formulation

We consider a CR network which shares the spectrum resource with a primary network, as illustrated in Fig. 1. Similar system models have been considered in [11], [12]. The primary network consists of a primary base station (PBS) that transmits signals to a single primary user. The secondary cognitive network has a single cognitive base station (CBS), equipped with M antennas, serving K secondary users. The k-th user is equipped with N_k antennas. Throughout this paper, we assume that $M << K$ and that the PBS and the primary user are equipped with a single antenna. Due to the sharing of the same frequency band, the received signal at the primary user is interfered by the signals transmitted from CBS. Similarly, the received signals at the secondary users are interfered by the signal transmitted from the PBS.

Assume that in one time slot, a block of information symbols $\mathbf{s} = [s_1, s_2, \cdots, s_K]^{\mathcal{T}}$ are sent from the CBS in which s_k, $k = (1, \cdots, K)$ is the desired signal for user k. We assume that $\mathbf{s}$ contains uncorrelated unit-power entries. With a proper power loading and beamforming (which will be specified later), the transmit signal is given by

$$\mathbf{x} \;=\; \mathbf{WPs}, \tag{1}$$

where $\mathbf{W} = [\ \mathbf{w}_1 \quad \mathbf{w}_2 \quad \cdots\ \mathbf{w}_K\ ]$ denotes the transmit beamforming matrix, with $\mathbf{w}_k$ being a $(M \times 1)$ vector. Likewise, $\mathbf{P} = \text{diag}\{\sqrt{P_1}, \cdots, \sqrt{P_K}\}$ accounts for power loading.

The received signal at the k-th cognitive user is given by

$$\mathbf{y}_k \;=\; \sqrt{\eta_k}\mathbf{H}_k\mathbf{x} + \sqrt{P_p}\mathbf{g}_k s_p + \mathbf{n}_k, \tag{2}$$

where η_k characterizes the path loss due to the distance between the CBS and the k-th user, $\mathbf{H}_k \in \mathbb{C}^{N_k\times M}$ denotes the channel from the CBS to the k-th cognitive user. The (n,m)-th entry $H_k(n,m) \sim \mathcal{CN}(0,1)$ is i.i.d. complex Gaussian with zero mean and unit variance and represents the complex channel gain from the transmit antenna m to receive antenna n of the k-th cognitive user. P_p denotes the transmitted power of the primary user. $\mathbf{g}_k$ represents the $(N_k \times 1)$ channel vector between the PBS and the k-th cognitive user, and s_p represents the transmitted signal from the PBS. Finally, $\mathbf{n}_k$ is a $(N_k \times 1)$ vector of additive noise whose entries are i.i.d. complex Gaussian with zero mean and variance σ_k^2.

For the sake of simplicity, we first restrict our discussion to the case of $N_k = 1$, $(k = 1, \cdots, K)$, i.e., we assume a single antenna at all receivers. The case of $N_k > 1$, i.e., MIMO, will be detailed in Section V. Then, we can rewrite (2) as

$$y_k = \sqrt{\eta_k}\mathbf{h}_k^{\mathcal{T}}\mathbf{x} + \sqrt{P_p} g_k s_p + n_k, \quad (3)$$

where $\mathbf{h}_k$ is a $(M \times 1)$ vector and denotes the channel link between the CBS and the k-th cognitive user.

The received signal at the primary user is given by

$$y_p = \sqrt{P_p} g_p s_p + \mathbf{h}_p^{\mathcal{T}}\mathbf{x} + n_p, \quad (4)$$

where g_p denotes the channel between the primary user and the PBS, $\mathbf{h}_p$ is a $(M \times 1)$ vector representing the channel between the CBS and the primary user and n_p denotes the additive noise and is assumed to be i.i.d. complex Gaussian distributed with zero mean and variance σ_p^2.

Note that y_k, $(k = 1, \cdots, K)$ are physically distributed across the K cognitive users. Multi-user decoding is not feasible. Hence, each user treats the signals intended for other users as interference. Then, the signal to interference plus noise ratio (SINR) of the k-th cognitive user is

$$\text{SINR}_k = \frac{|\mathbf{h}_k^{\mathcal{T}}\mathbf{w}_k|^2 P_k \eta_k}{\sum_{q \neq k} |\mathbf{h}_k^{\mathcal{T}}\mathbf{w}_q|^2 P_q \eta_k + |g_k|^2 P_p + \sigma_k^2}. \quad (5)$$

In order to allow the cognitive users to share the spectrum with the primary user, we should investigate appropriate power and beamforming weights to distribute them among the users so that the total cognitive system throughput is maximized, and the interference created to the primary user is as small as possible. In particular, by applying an appropriate scheduling scheme, one will be able to select the cognitive users that have less effect on the primary user, i.e., create less interference to it. The power loading problem among the K cognitive users, based on the proposed opportunistic spectrum sharing goal, can be written as

$$\max_{\mathbf{P},\mathbf{W}} \quad \sum_{k=1}^{K} \log\left(1 + \text{SINR}_k\right) \quad (6)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} P_k \leq P_t, \quad (7)$$

where P_t is the total power.

When the number of the total users K goes to infinity, it is not easy to solve the above problem due to the high complexity. The base station can schedule its transmission to only a set of users in order to decrease the complexity. The set of users should be selected with favorable channel conditions to improve the system throughput. On one hand, multi-user diversity is able to increase channel magnitudes by choosing the user with a good channel condition. On the other hand, multi-user diversity is able to increase the freedom by choosing the user with a good spatial separation. It was shown in [13] that for a system with a large number of users, i.e., $K >> M$, a maximum downlink throughput can be achieved by simply serving M users simultaneously. Hence, selecting only M users is able to reduce the computational complexity of the system, which is crucial especially with a large number of users. In our work, since a large number of secondary users is considered, we aim to select a set $\mathcal{S}$ of size M cognitive users in order to satisfy the following two goals: 1) to maximize the total cognitive system throughput; and 2) to protect the primary system from harmful interference.

We next propose an opportunistic spectrum sharing approach that enables the opportunistic use of the cognitive users to coexist with the primary user in order to improve the spectrum efficiency. In particular, the proposed approach is able to select the users to improve the cognitive system throughput without creating harmful interference to the primary user. Specifically, the proposed approach can be described by the following

- *Cognitive user selection algorithm*: This step aims at scheduling $|\mathcal{S}| = M$ cognitive users among a total of K users. The M users are selected in order to maximize the cognitive system throughput and minimize the interference caused to the primary user. This step is the concern of this paper, and it will be developed throughout the remaining parts of the paper, based on different scenarios and different channel conditions.
- *ZFB*: This step aims to cancel the interference among cognitive users when they are served by the CBS at the same band simultaneously. Furthermore, ZFB plays a critical role in limiting the interference to the primary user in our proposed user selection, which will be specified later.

After selecting a set $\mathcal{S}$ users and applying ZFB to these selected candidates, the optimal solution P_k to the problem in (6), after restricting ourselves to M selected users, can be easily found by a spectral water-filling.

III. Opportunistic Spectrum Sharing

Since a large number of users compared to the number of antennas at the CBS is considered, a subset of users should be selected in order to decrease the complexity of the system. Multi-user selection has been recently studied in the literature [14]–[18]. In particular, the base station has to schedule its transmission to the users with the best channel conditions in order to improve the system performance and satisfy its requirements. To achieve high rates, dirty paper coding (DPC) techniques can be deployed [19]. However, DPC is difficult to implement in practice due to its high complexity, and in particular when the number of users is large. Recently, a ZFB scheme has been considered in [14] for downlink setup under a sum-power constraint. In this scheme, a set of semi-orthogonal users to be served is selected so as to maximize the sum-rate. In [15], a near-orthogonal set of channel vectors which meet certain system requirements is selected. In [16], by applying singular value decomposition (SVD) to the users' channel matrices, only the eigenvectors whose corresponding singular values are larger than a given threshold are selected. Among these candidates, the users that are near-orthogonal to each other are then chosen. Hence, the complexity of the user selection scheme is reduced compared to that in [14]. In [17], a multi-user scheduling method was proposed. This method allocates independent information streams from all M transmit antennas to the M most favorable users with the highest SINR. In [18], a greedy user selection and linear precoding strategy, which achieve the same scaling as the sum capacity of the MIMO broadcast channels, have been proposed.

Unlike conventional cellular networks, in cognitive networks, one should deal with not only the interference between the cognitive users, but also the interference to the primary user. For this purpose, we propose a $\{\delta_p, \delta_c\}$-orthogonal user selection along with ZFB. In this section, we restrict ourselves to the case of a MISO cognitive system, i.e., $N_k = 1,\ k = 1, \cdots, K$. The proposed algorithm, along with the ZFB, is able to maximize the cognitive system throughput and minimize the amount of interference to the primary user. In particular, the algorithm selects a set of cognitive users that cause less interference to each other and less interference to the primary user. Furthermore, ZFB will totally null the interference between the cognitive users.

A. $\{\delta_p, \delta_c\}$-orthogonal user selection

Definition: Let

$$\Delta(\mathbf{h}_i, \mathbf{h}_j) \triangleq \frac{|\mathbf{h}_i^{\mathcal{H}}\mathbf{h}_j|}{||\mathbf{h}_i||||\mathbf{h}_j||}.$$

Users i and j are called δ-orthogonal if and only if

$$\Delta(\mathbf{h}_i, \mathbf{h}_j) \leq \delta. \tag{8}$$

Our proposed $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm can then be described in Algorithm 1. The algorithm can be explained as follows. In the first step, cognitive user k, $(k = 1, \cdots, K)$ is selected when $\Delta(\mathbf{h}_p, \mathbf{h}_k) \leq \delta_p$ is satisfied. The set of the selected users is denoted by $\mathcal{Q}_1$. Let J_1 be the cardinality of this set, i.e., $J_1 = |\mathcal{Q}_1|$. The first user is then chosen to have the maximal channel norm among those J_1 users in the set $\mathcal{Q}_1$. In the second step, if $|\mathcal{S}|$, the number of the selected users, satisfies $|\mathcal{S}| < M$, the algorithm proceeds to the selection part. Otherwise, the algorithm is stopped. In particular, the i-th selected user is chosen as the one having maximum value of the channel gain among the users satisfying δ_c orthogonality condition. Finally, we obtain a set $\mathcal{S}$ of cognitive users that are δ_c-orthogonal to one another and δ_p-orthogonal to the primary user, with relatively large channel gains. The proposed algorithm minimizes the interference between the cognitive users and the primary user, by guaranteeing that any cognitive user to be near-orthogonal to the primary user, by a factor of δ_p. Moreover, it allows any two cognitive users to be near-orthogonal to each other, by a factor of δ_c. This step is then able to select the users that can make the capacity as large as possible.

Algorithm 1 $\{\delta_p, \delta_c\}$-orthogonal User Selection

Step 1: δ_p-orthogonal user selection

1) $\mathcal{S} = \emptyset$.
2) The candidates of users satisfying δ_p-orthogonality are denoted by
$$\mathcal{Q}_1 = \{k|\Delta(\mathbf{h}_p, \mathbf{h}_k) \leq \delta_p, k = 1, \cdots, K\}. \tag{9}$$
3) The first selected user is determined by
$$\mathcal{S}(1) \quad = \quad \arg\max_{k \in \mathcal{Q}_1} ||\mathbf{h}_k||. \tag{10}$$

Step 2: δ_c-orthogonal user selection

1) $i = 1$;
2) While $i < M$,
   a) $i = i + 1$;
   b) The candidates of users satisfying δ_c-orthogonality are denoted by
   $$\mathcal{Q}_i = \{k|\Delta(\mathbf{h}_{\mathcal{S}(i-1)}, \mathbf{h}_k) \leq \delta_c, \forall k \in \mathcal{Q}_{i-1}\}. \tag{11}$$
   c) The i-th selected user is determined by
   $$\mathcal{S}(i) \quad = \quad \arg\max_{k \in \mathcal{Q}_i} ||\mathbf{h}_k||. \tag{12}$$

End

B. Zero-forcing beamforming

Transmit antenna arrays have great potential to control co-channel interference and achieve high throughput in wireless systems. In scenarios where antenna arrays are used at the transmitters, the beam-pattern of each antenna can be adjusted to minimize the interference to the undesired receivers. Transmit beamforming has been extensively considered for cellular systems. In [20], an algorithm was proposed to jointly search for a set of a feasible transmit beamforming weight vector and downlink transmit power allocations so that the SINR at each link is greater than a target value. Moreover, transmit beamforming has been exploited as a strategy that can serve many users at similar throughput as DPC but with lower complexity [21].

In this paper, we utilize the simple principle of ZFB that nulls interference between cognitive data streams. Transmit beamforming weights can be easily found by inverting the channel matrix of the selected users. Using such a scheme, the mutual interference among the selected users can be nulled by selecting appropriate beamforming weight vectors according to the principle of ZFB which transforms the broadcast channels into parallel, independent and orthogonal sub-channels. In particular, beamforming vectors are selected so that they satisfy the interference-free condition. That is, $\mathbf{h}_k^{\mathcal{H}}\mathbf{w}_j = 0$ for $j \neq k$. In order to obtain interference-free between the cognitive users, we get the beamforming weights vectors $\mathbf{W}(\mathcal{S})$ by inverting the channel matrix of the selected users $\mathbf{H}(\mathcal{S})$. Then, the channel matrix can be written as

$$\mathbf{W}(\mathcal{S}) = \mathbf{H}(\mathcal{S})^{\dagger} = \mathbf{H}(\mathcal{S})^{\mathcal{H}}(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^{\mathcal{H}})^{-1}. \tag{13}$$

The ZFB is able to cancel the interference between the cognitive users and hence maximize the throughput of the cognitive system. Moreover, ZFB combined with our proposed

δ_p-user selection can greatly reduce the interference. From (4), the interference power caused by the cognitive user at the primary user is

$$\begin{aligned} I &= |\mathbf{h}_p^{\mathcal{T}}\mathbf{x}|^2 \\ &= \sum_{k\in\mathcal{S}} |\mathbf{h}_p^{\mathcal{T}}\mathbf{w}_k|^2 P_k. \end{aligned} \quad (14)$$

It can be seen from (14) that the interference will be reduced after choosing the appropriate beamforming weights.

IV. Capacity Analysis

A. Multi-user diversity gain

Multi-user diversity gain is related to the size of the set from which the cognitive user k is chosen. In the proposed system, the users are firstly selected from a large set of K users. The user selection (Step 2) will then become less complex since the number of users is reduced from K to J_1. Since the proposed $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm consists of two steps, the analysis of the multi-user gain reduction will be done for both steps of the user selection.

1) δ_p-orthogonal user selection: In the δ_p-orthogonal user selection, the cognitive user candidates are chosen based on the δ_p-orthogonality. For user k, the probability of this user being selected by the δ_p-orthogonal user selection algorithm is [See Appendix I]

$$\text{Prob}\{k \in \mathcal{Q}_1\} = 1 - (1-\delta_p^2)^{M-1}. \quad (15)$$

To find the cardinality of $\mathcal{Q}_1$, i.e., J_1, we apply the law of large numbers. For a large number of users K, J_1 can be approximated as

$$\begin{aligned} J_1 &\approx \lfloor K \ \text{Prob}\{k \in \mathcal{Q}_1\}\rfloor \\ &\approx \lfloor K\big(1-(1-\delta_p^2)^{M-1}\big)\rfloor. \end{aligned} \quad (16)$$

2) δ_c-orthogonal user selection: In this step, the cognitive user k is chosen based on the δ_c-orthogonality defined in (11). For user k, the probability of this user being selected by the δ_c-orthogonal user selection algorithm is [See Appendix II]

$$\text{Prob}\{k \in \mathcal{Q}_i\} = I_{\delta_c^2}(i, M-i), \quad (17)$$

where $I_x(a,b)$ denotes the regularized incomplete beta function [22]. For a large number of users K, J_i can be approximated as

$$\begin{aligned} J_{i+1} &\approx \lfloor J_1 \ \text{Prob}\{k \in \mathcal{Q}_{i+1}\}\rfloor \\ &\approx \lfloor J_1 I_{\delta_c^2}(i, M-i)\rfloor, \quad i = 1, \cdots, M-1. \end{aligned} \quad (18)$$

To guarantee good performance when the proposed user selection scheme is used, a large number of users is desired. However, a large number of users induces high complexity. In Section VI, we will analyze the complexity caused by the proposed scheme.

3) Choices of δ_p and δ_c: Our aim here is to find the values of δ_p and δ_c that the system has to set for a better user selection. At the end of the δ_p-orthogonal user selection, we get J_1 instead of K users. In order to make the selection of M users possible, we should guarantee that $J_1 \geq M$. Hence, based on (16), δ_p should satisfy the following condition

$$\begin{aligned} & K\big(1-(1-\delta_p^2)^{M-1}\big) \geq M \\ \Longleftrightarrow \quad & \delta_p \geq \sqrt{1-\left(1-\frac{M}{K}\right)^{\frac{1}{M-1}}}. \end{aligned} \quad (19)$$

In the δ_c-orthogonal user selection and in order to guarantee that, at the end of the selection step, $J_M = |\mathcal{Q}_M| \geq 1$, the following condition, which can be derived from (17), must be satisfied

$$J_1 \cdot I_{\delta_c^2}(M-1, 1) \quad \geq \quad 1. \quad (20)$$

For example, for $M = 2$, $\delta_p = 0.7$ and $K = 100$, using (20), we have $I_{\delta_c^2}(1,1) \geq 0.02$. Based on the definition of the regularized incomplete beta function, we obtain $\delta_c \geq 0.141$. Hence, once the number of users K and transmit antennas M are known, we can set the lower bounds of the thresholds δ_p and δ_c.

B. Lower-bound performance

Since it is not trivial to perform the exact capacity analysis of the $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm, we intend to derive a lower bound of the capacity as follows. With the $\{\delta_p, \delta_c\}$-orthogonal user selection, and equal power allocation along with ZBF, the cognitive system sum-rate can be obtained from (6) as

$$R \quad = \quad \sum_{k\in\mathcal{S}} \log\left(1 + |\mathbf{h}_k^{\mathcal{T}}\mathbf{w}_k|^2 \frac{\eta_k P_t}{M(|g_k|^2 P_p + \sigma_k^2)}\right). \quad (21)$$

We define the channel alignment by $\cos^2\phi_k = |\bar{\mathbf{h}}_k^{\mathcal{T}}\mathbf{w}_k|^2$, where $\bar{\mathbf{h}}_k$ is the normalized channel gain, i.e. $\bar{\mathbf{h}}_k = \frac{\mathbf{h}_k}{||\mathbf{h}_k||}$. Then, (21) becomes

$$R \quad = \quad \sum_{k\in\mathcal{S}} \log\left(1 + ||\mathbf{h}_k||^2 \frac{\eta_k P_t \cos^2\phi_k}{M(|g_k|^2 P_p + \sigma_k^2)}\right). \quad (22)$$

Using the result in [14], it follows that

$$\begin{aligned} \cos^2\phi_k = |\bar{\mathbf{h}}_k^{\mathcal{T}}\mathbf{w}_k|^2 \quad &\geq \quad \frac{(1+\delta_c)(1-(M-1)\delta_c)}{(1-(M-2)\delta_c)} \\ &\triangleq \quad \cos^2\phi. \end{aligned} \quad (23)$$

We can then get the lower bound of the sum-rate as

$$R \geq \sum_{k\in\mathcal{S}} \log\left(1 + ||\mathbf{h}_k||^2 \cos^2\phi \frac{\eta_k P_t}{M(|g_k|^2 P_p + \sigma_k^2)}\right). \quad (24)$$

Define

$$\bar{\gamma}_k \quad \triangleq \quad \frac{\eta_k P_t}{|g_k|^2 P_p + \sigma_k^2} \cos^2\phi, \ \ k = 1, \cdots, K, \quad (25)$$

$$\alpha_k \quad \triangleq \quad \frac{1}{M} ||\mathbf{h}_k||^2, \ \ k = 1, \cdots, K, \quad (26)$$

and denote (i) the index of the i-th selected user by our algorithm. Then, (24) can be rewritten as

$$\begin{aligned} R &\geq \sum_{i=1}^{M} \log\left(1+\bar{\gamma}_{(i)} \cdot \alpha_{(i)}\right), \\ &= \sum_{i=1}^{M} \log\left(1+\gamma_i\right) \end{aligned} \tag{27}$$

where $\bar{\gamma}_{(i)}$ and $\alpha_{(i)}$ denote the average and the instantaneous SINR of the i-th selected user, respectively, and $\gamma_i \triangleq \bar{\gamma}_{(i)} \cdot \alpha_{(i)}$. In the following, we will calculate the probability density function (PDF) of γ_i to get the average sum-rate.

We begin by noting that the i-th user is selected as the best one from J_i users according to the instantaneous channel gain α_k, i.e.,

$$\alpha_{(i)} = \max_{k\in\mathcal{Q}_i} \alpha_k. \tag{28}$$

The PDF of $\alpha_{(i)}$ can be computed using order statistics [27] on the assumption of identical distribution of α_k for all users, as

$$f_{\alpha_{(i)}}(\gamma) = J_i \cdot f_\alpha(\gamma) \cdot F_\alpha(\gamma)^{J_i-1}, \tag{29}$$

where $f_\alpha(\gamma)$ and $F_\alpha(\gamma)$ denote the PDF and the cumulative distribution function (CDF) of α_k, respectively. Specifically, $f_\alpha(\gamma)$ is given by

$$f_\alpha(\gamma) = \frac{M^M}{(M-1)!}\gamma^{M-1}e^{-M\gamma},\ \gamma \geq 0. \tag{30}$$

Assuming identical small-scale fading statistics across the mobile users, the scheduler would be able to provide equal chance to all users for packet transmission regardless of the average users' SINR. Hence, since the scheduler has no consideration of the average SINR $\bar{\gamma}_k$, under the assumption of identical small-scale fading statistics across the user candidates, the PDF $\bar{\gamma}_{(i)}$ of the users, having equal access time, is given by

$$f_{\bar{\gamma}_{(i)}}(\gamma) = \frac{1}{J_i}\sum_{k\in\mathcal{Q}_i} \delta(\gamma-\bar{\gamma}_k). \tag{31}$$

Therefore, the PDF of γ_i, $\gamma_i = \bar{\gamma}_{(i)} \cdot \alpha_{(i)}$, is given by [28]

$$\begin{aligned} f_{\gamma_i}(\gamma) &= \int_{-\infty}^{\infty} \frac{1}{|x|} f_{\bar{\gamma}_{(i)}}(x) f_{\alpha_{(i)}}(\frac{\gamma}{x})dx \\ &= \frac{1}{J_i}\sum_{k\in\mathcal{Q}_i}\frac{1}{\bar{\gamma}_k} f_{\alpha_{(i)}}(\frac{\gamma}{\bar{\gamma}_k}). \end{aligned} \tag{32}$$

Using (29), the expression of the PDF of γ_i in (32) becomes

$$\begin{aligned} f_{\gamma_i}(\gamma) &= \sum_{k\in\mathcal{Q}_i} \frac{M^M}{\bar{\gamma}_k(M-1)!}\left(\frac{\gamma}{\bar{\gamma}_k}\right)^{M-1} e^{-\frac{M\gamma}{\bar{\gamma}_k}} \\ &\times \left(1-e^{-\frac{M\gamma}{\bar{\gamma}_k}}\sum_{m=0}^{M-1}\frac{(M\gamma/\bar{\gamma}_k)^m}{m!}\right)^{J_i-1}. \end{aligned} \tag{33}$$

Then, the lower bound of the average capacity can be expressed as

$$\begin{aligned} C &\geq \sum_{i=1}^{M}\int_0^\infty \log(1+\gamma) f_{\gamma_i}(\gamma)d\gamma, \\ &= \sum_{i=1}^{M}\sum_{k\in\mathcal{Q}_i}\frac{M^M}{\bar{\gamma}_k^M(M-1)!}\int_0^\infty \log(1+\gamma)e^{-\frac{M\gamma}{\bar{\gamma}_k}}\gamma^{M-1} \\ &\times \left(1-e^{-\frac{M\gamma}{\bar{\gamma}_k}}\sum_{m=0}^{M-1}\frac{(M\gamma/\bar{\gamma}_k)^m}{m!}\right)^{J_i-1} d\gamma. \end{aligned} \tag{34}$$

V. MIMO Cognitive Networks

In the previous sections, we have considered the case where each cognitive receiver is equipped with only one antenna. As an extension of the multiple-input/single-output (MISO) case, we consider multiple antennas at each of K cognitive users in this section.

A. MIMO receiver design

We suppose that the k-th user has N_k receive antennas. The receiver processing strategy adopted in this section will be explained in the following. The SVD of the channel matrix $\mathbf{H}_k$ is given by

$$\mathbf{H}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{V}_k^{\mathcal{H}},\ k=1,\cdots,K \tag{35}$$

where $\mathbf{\Lambda}_k$ is an $N_k \times M$ diagonal matrix containing the singular values of $\mathbf{H}_k$, $\mathbf{U}_k$ and $\mathbf{V}_k$ are $N_k \times N_k$ and $M \times M$ unitary matrices, respectively. The received vector for the k-th user $\mathbf{y}_k$ in (2) can be rewritten as

$$\mathbf{y}_k = \sqrt{\eta_k}\mathbf{H}_k\mathbf{x} + \tilde{\mathbf{n}}_k,\ k=1,\cdots,K \tag{36}$$

where $\tilde{\mathbf{n}}_k = \sqrt{P_p}\mathbf{g}_k s_p + \mathbf{n}_k$. Let $\mathbf{u}_{k,n}$ be the n-th column of $\mathbf{U}_k$. By multiplying both sides of (36) by $\mathbf{u}_{k,n}^{\mathcal{H}}$, we can get

$$r_{k,n} = \mathbf{f}_{k,n}\mathbf{x} + w_{k,n},\qquad k=1,\cdots,K,\ n=1,\cdots,N_k \tag{37}$$

where

$$r_{k,n} = \mathbf{u}_{k,n}^{\mathcal{H}}\mathbf{y}_k,\ \mathbf{f}_{k,n} = \sqrt{\eta_k}\sqrt{\lambda_n(k)}\mathbf{v}_{k,n}^{\mathcal{H}},\ \text{and } w_{k,n} = \mathbf{u}_{k,n}^{\mathcal{H}}\tilde{\mathbf{n}}_k. \tag{38}$$

In the above equations, $\mathbf{v}_{k,n}$ denotes the n-th column of $\mathbf{v}_k$ and $\sqrt{\lambda_n(k)}$ is the n-th singular value of $\mathbf{H}_k$ corresponding to $\mathbf{v}_{k,n}$.

After the SVD, the resulting channel can be viewed as a MIMO broadcast channel with $\sum_{k=1}^K N_k$ single antenna users. In such scenario, different possibilities can be considered at the receiver side [14], [30]. The first case is to treat each receive antenna as an independent user. In this case, we will have $\sum_{k=1}^K N_k$ single antenna receivers. Therefore, the k-th user should feedback N_k times the amount of information fed back by a system with a single antenna. The second case is to assign at most one beam to each user. In such case, each user just feeds back its channel gain corresponding to the selection scheme and the corresponding antenna index n. The third case is to assign multiple beams to each user. In this case, we either assign N_k beams to the k-th user or no beams at all. As shown

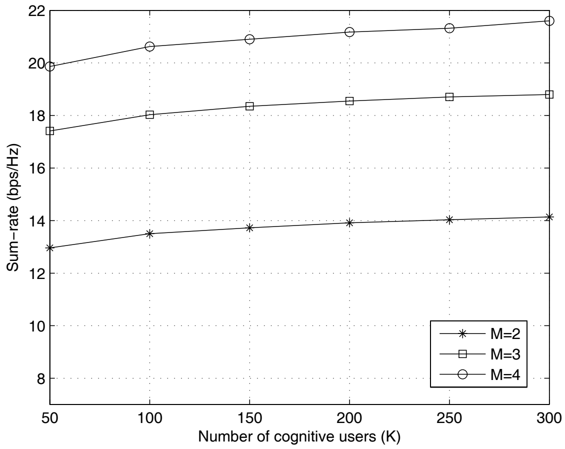

Fig. 2. Sum-rate versus the number of users for different number of transmit antennas M. $N = 1$, $\delta_p = 0.8$, $\delta_c = 0.6$, and $P_t = 20$ dB.

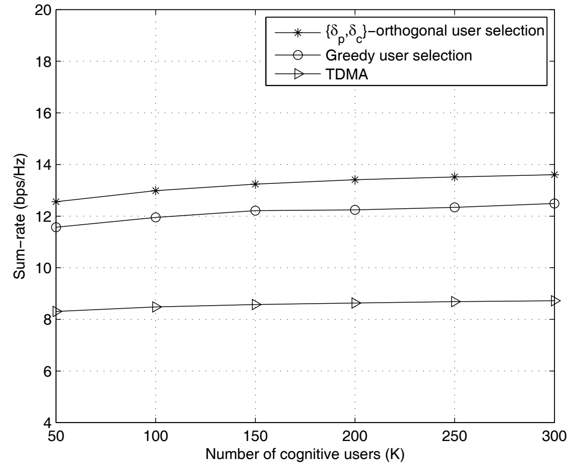

Fig. 3. Sum-rate of $\{\delta_p, \delta_c\}$-orthogonal user selection,TDMA, and greedy selection versus the number of users. $M = 2$, $\delta_p = 0.8$, $\delta_c = 0.8$, and $P_t = 20$ dB.

in [30], the first case is effectively the same as the one having $\sum_{k=1}^{K} N_k$ users with a single receive antenna. The second case is a generalization of the case with $N = 1$, and it was shown that its analysis is very similar to the previous case. On the other hand, the third case is quite different from the previous two and requires more efforts to be analyzed. In terms of the amount of feedback, clearly the first case requires N times more feedback than that of the second and third cases.

In [14], based on SVD, the system is viewed as a MIMO channel with $\sum_{k=1}^{K} N_k$ SISO sub-channels and the previously proposed selection algorithm can be applied to those single antenna users. However, in this method, the receive antennas can be selected from the same user, which is not fair because other users have no chance to share the spectrum resource. To guarantee fairness between users and multi-user diversity, all users should have the chance to be selected. In [30], for the MIMO case, each receive antenna at the receiver is treated as an individual user, then the system will be equivalent to $\sum_{k=1}^{K} N_k$ single antenna receivers. Based on the aforementioned receiver, it has been also shown that each receive antenna should be treated as an individual user in order to achieve the largest throughput. Therefore, the K-user $M \times N_k$ system is converted to a $(\sum_{k=1}^{K} N_k)$-user $M \times 1$ system.

In the following, we propose a MIMO downlink scheduling algorithm based on receive antenna selection in order to further reduce the complexity of the user selection.

B. Receive antenna selection

The key idea behind our selection algorithm is that each user selects at the first stage the most favorable receive antenna. The receive antenna is selected based on the channel conditions, i.e., the best channel quality. In particular, by applying SVD to all users' channel matrices, only the eigenvectors whose corresponding singular values are the maximum are considered. Then, among these candidate eigenvectors, the algorithm chooses a set of a size M users which are nearly orthogonal to each other and nearly orthogonal to the primary user based on the $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm. Using such a method, the amount of feedback and the size of selecting space for selecting the candidates is reduced. Let $\mathcal{A}$ denote the set of the selected receive antennas. Our proposed scheme of the antenna selection is described in Algorithm 2.

After the selection of K antennas, we will have K single antenna users, and the same method of selection in Section III is applied. In particular, we apply Algorithm 2 on the K selected antennas. Let p and q be the indices for the selected

Algorithm 2 Receive Antenna Selection

1) Initialization: $\mathcal{A} = \emptyset$.
2) For $k = 1 : K$,
   1) Using SVD, the k-th cognitive user calculates the eigenvectors and singular values of its channel matrix based on (35). Then, it sends back the maximum singular values, along with their corresponding "right" eigenvectors, to the CBS.
   2) Select the receive antenna n such that

$$n = \arg\max \lambda_n(k),\ \mathcal{A} = \mathcal{A} \cup \{n\}. \quad (39)$$

End

receive antenna and its corresponding user, respectively. After the antenna selection, the selected coordinate matrix can be written as

$$\mathbf{F} = [\mathbf{f}_{q_1,p_1}^{\mathcal{T}}\ \mathbf{f}_{q_2,p_2}^{\mathcal{T}}\ \cdots\ \mathbf{f}_{q_M,p_M}^{\mathcal{T}}]^{\mathcal{T}}, \quad (40)$$

where $\mathbf{f}$ is given in (38). After applying ZFB, $\mathbf{W} = \mathbf{F}^{\dagger}$, using (37), the received signal is given by

$$r_{q_l,p_l} = \sqrt{\eta_l}\sqrt{\lambda_{p_l}(q_l)}\sqrt{P_l}s_l + w_{q_l,p_l},\ l = 1, \cdots, M$$

where l is the index of the received signal. Obviously, applying ZFB is of great interest of decomposing the channel to M interference-free sub-channels among users.

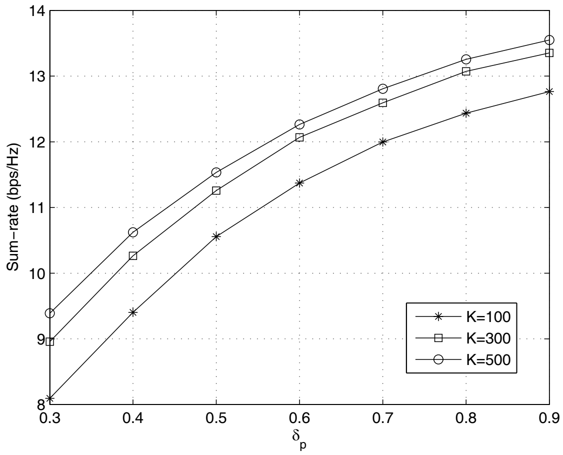

Fig. 4. Sum-rate versus δ_p at various number of users K. $M = 2$, $N = 1$, $\delta_c = 0.8$, and $P_t = 20$ dB.

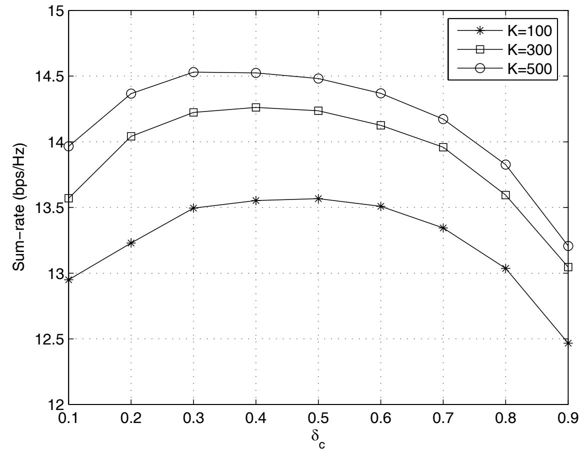

Fig. 5. Sum-rate versus δ_c at various number of users. $M = 2$, $N = 1$, $\delta_p = 0.8$, and $P_t = 20$ dB.

VI. Performance Analysis

A. Simulation results

In order to evaluate the performance of the proposed algorithm, the sum-rate performance is considered. Fig. 2 shows the sum-rate of cognitive users as a function of number of users for $\delta_p = 0.8$, and $\delta_c = 0.6$. We set $M = 2,\ 3$ and 4, while $N_k = 1$ $k = 1, \cdots, K$. When the number of antennas at the base station is large, it is obvious that the sum-rate of the cognitive users increases. In fact, the capacity increases linearly with $\min\{M, K\}$. Therefore, we can conclude that by serving just M users, one can achieve a large throughput [13].

Throughout the remaining simulation studies, we set the number of transmit antennas to be $M = 2$. In Fig. 3, the sum-rate of the cognitive system, averaged over the channel distributions, under the proposed algorithm, a greedy user scheme, and the TDMA scheme are presented in terms of the number of users. With the TDMA scheme, the user with the best channel condition is scheduled, while using the greedy scheme, we allow the M users that have the best channel conditions. The parameters used for the simulation are set as $N_k = 1$ $(k = 1, \cdots, K)$, $\delta_p = 0.8$ and $\delta_c = 0.8$. The plots show that the performance of the $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm is better than the one performed under the TDMA scheme. Besides, our proposed algorithm performs better than the greedy scheme. In fact, this is expected because with the greedy selection, the users are selected based on their best channel condition. In this case, the interference created to the primary user may be harmful, and as a result, it will degrade the cognitive system performance.

The sum-rate of cognitive users as a function of δ_p for $N_k = 1$ $(k = 1, \cdots, K)$, $\delta_c = 0.8$, and $P_t = 20$ dB is illustrated in Fig. 4. When δ_p is large, the system sum-rate increases, because the probability that more users are scheduled is high. When δ_p is small, the multi-user diversity decreases. Hence, we can observe from the curve that the sum-rate is reduced. Moreover, it can be noticed that the sum-rate increases with the number of cognitive users.

In Fig. 5, the sum-rate of cognitive users as a function of δ_c is presented. We set $N_k = 1$ $(k = 1, \cdots, K)$, $\delta_p = 0.8$, and $P_t = 20$ dB. A close observation of this figure indicates that for small values of δ_c, the sum-rate grows with the increase of δ_c until hitting a peak value. When δ_c is equal to 0, users can not be selected because finding users that are totally orthogonal is not possible in a real environment. Hence, this clearly demonstrates the importance of the proposed $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm. As δ_c increases from zero and as long as δ_c is small, more candidate users are able to guarantee the δ_c-orthogonality. As a result, a better throughput is achieved by exploiting multi-user diversity. However, when δ_c becomes too large, the throughput decreases. This can be explained as follows. After the first step of selection, the scheduled users are δ_p-orthogonal to the primary user with large channel gains. However, with large δ_c, the users that are highly correlated to each other may be selected, and this degrades the total system throughput. From Fig. 5 and based upon extensive simulations, we found that the best range for δ_c in terms of high throughput is the range of $[0.2, 0.4]$. This finding is consistent with the results of [14],

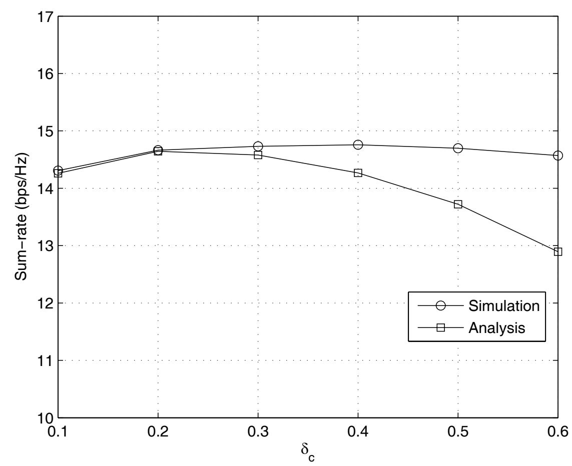

Fig. 6. Numerical and analytical sum-rate versus δ_c. $K = 800$, $M = 2$, $N = 1$, $\delta_p = 0.8$, and $P_t = 20$ dB.

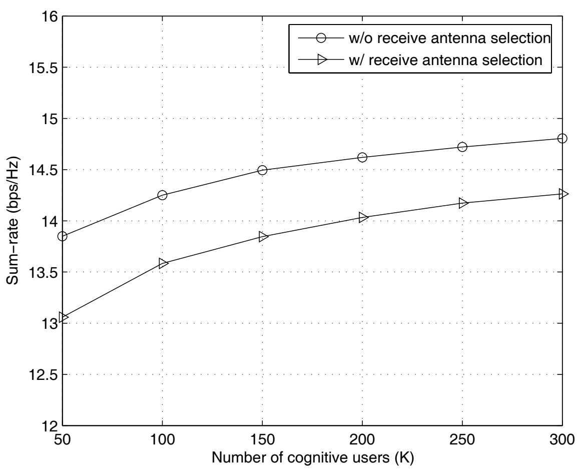

Fig. 7. Sum-rate versus the number of users with/without receive antenna selection. $M = 2$, $N = 3$, $\delta_p = 0.8$, $\delta_c = 0.4$, $P_t = 20$ dB.

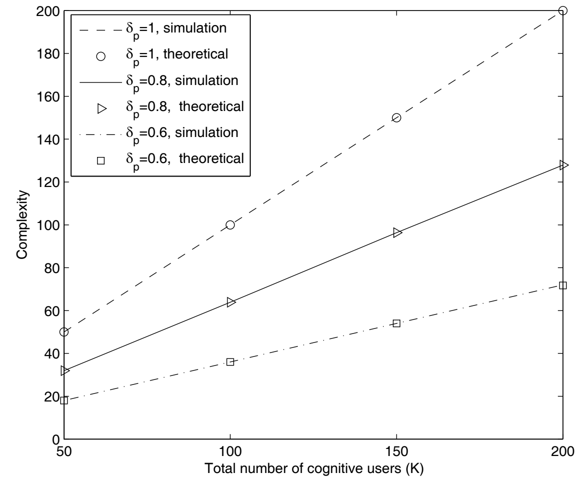

Fig. 8. Complexity versus the number of users for different δ_p.

where a semiorthogonal user selection (SUS) was used to select a group of candidate users in wireless networks. The next interesting question is how to select the "best" value of δ_c within the optimal range $[0.2, 0.4]$ to achieve a high throughput. For that, we shall resort to the lower bound derived in Section IV.

In Fig. 6, we use the same simulation settings used for Fig. 5 and $K = 800$. The figure shows the throughput of the cognitive system when δ_c is selected in the optimal range. The sum-rate of the cognitive system generated by simulation is compared to the theoretical lower bound of the average capacity in (34). Again from Fig. 6, we observe that the optimal range of δ_c can be chosen to be from 0.2 to 0.4. More importantly, note that our bound is loose when δ_c is large. However, this is not an issue since the range of interest is $[0.2, 0.4]$ where our lower bound is relatively tight. In particular, the bound can predict the value of δ_c that can achieve the highest throughput. As a result, we can conclude that the proposed bound can prove to be a useful tool for finding the optimal value of δ_c without resorting to extensive simulations.

Fig. 7 compares the sum-rate throughput of the proposed algorithm with and without receive antenna selection, when $\delta_p = 0.8$, $\delta_c = 0.4$ and multiple receive antennas are considered at the receiver. It can be seen that the $\{\delta_p, \delta_c\}$-orthogonal user selection with receive antenna selection gives a little worse performance than the $\{\delta_p, \delta_c\}$-orthogonal user selection when no receive antenna selection is considered.

We define the scheme without antenna selection by a scheme where the system has $\sum_{k=1}^{K} N_k$ single antenna users. Since in the scheduling schemes, multi-user diversity is exploited, the system with $\sum_{k=1}^{K} N_k$ cognitive users (without receive antenna selection) is intended to achieve better performance compared to the system with K users (with receive antenna selection). It will be shown later that a lower complexity in the user selection is achieved at the expense of this loss of throughput.

B. Complexity analysis

1) User selection algorithm: As it has been seen previously, the user selection consists of two steps: δ_p-orthogonal user selection and δ_c-orthogonal user selection. In the first step, for the selection strategy, we need K times inner product operations and $2K$ vector 2-norm calculations. Let $\mathcal{B}_1$ be the computational complexity of the first step, then,

$$\mathcal{B}_1 = \mathfrak{C}K, \tag{41}$$

where $\mathfrak{C}$ is a proportionality constant that corresponds to one inner product and two vector 2-norm calculations. We mean by computational complexity the number of calculations needed in our proposed user selection scheme, during the simulations.

In the second step, we need at $J_i = |\mathcal{Q}_i|$ inner product operations and $2M$ vector 2-norm calculations, according to the δ_c-orthogonality condition between any two candidate users, at the i-th iteration of the selection. We can write the computational complexity of running this step, $\mathcal{B}_2$, as

$$\mathcal{B}_2 = \mathfrak{C} \sum_{i=1}^{M-1} J_i. \tag{42}$$

Hence, the complexity $\mathcal{B}$ for the whole user selection process is as follows

$$\begin{aligned} \mathcal{B} &= \mathcal{B}_1 + \mathcal{B}_2 \\ &= \mathfrak{C}\Big(\sum_{i=1}^{M-1} J_i + K \Big). \end{aligned} \tag{43}$$

In the following, we point out the advantages of our proposed algorithm, in terms of complexity, compared to DPC:

- The complexity of DPC is presented by an optimization problem that can be solved by a sum-power iterative algorithm with high complexity [23], [24]. Hence, the proportionality constant for the sum-power iterative water-filling is much larger than $\mathfrak{C}$. Compared to the complexity of DPC, the complexity of our scheme $\mathcal{B}$ in (43) is reduced using the $\{\delta_p, \delta_c\}$-orthogonal user selection algorithm.
- To obtain the beamforming weight vectors, DPC requires many matrix multiplications and inversions along with

singular value decompositions [25]. However, using our proposed scheme, only one matrix inversion is needed to get the beam weight vectors.

- DCP needs to perform pre-coding that uses concatenated coding with high complexity [26]. For our scheme, because of the use of ZFB, no interference pre-substraction is required.

Obviously, our proposed user selection scheme has a lower complexity compared to DPC. Fig. 8 shows the complexity in terms of the number of cognitive users J_1 as a function of the total number of users in the cognitive network, for different values of δ_p. In the simulations, we use $M = 2$, $\delta_p = 0.6,\ 0.8,\ 1$. The numerical results are compared to the analytical ones found in (16). From the plots in Fig. 8, it can be observed that the simulation and the analytical results match well. Besides, we can easily notice from the figure that when the factor δ_p is smaller, the complexity decreases. In order to have lower complexity in the second step of the user selection, δ_p can be set to the bound in (19).

2) Receive antenna selection: The use of ZFB and the $\{\delta_p, \delta_c\}$-orthogonal user selection is able to decrease the complexity of the selection, as shown in the previous subsection. Moreover, the use of receive antennas selection can further decrease the complexity. We investigate the complexity of our proposed algorithm in terms of the amount of feedback required from the users to the CBS and the search complexity.

As can be observed in the proposed algorithm, only the eigenvectors that belong to $\mathcal{A}$, defined in (39), must be sent back to the base station, along with their corresponding singular values. In fact, $2M$ values should be fed back to the CBS for each eigenvector and singular value. In our algorithm, each cognitive user selects one receive antenna. Hence, the total number of real values to be fed back is $2MK$. As such, the amount of feedback is reduced compared to the one in [30], which is equal to $2NMK$.

Finally, we note that the proposed method is able to reduce the search complexity. In the step of receive antenna selection, only one eigenvector for each cognitive user is pre-selected. Therefore, the size of the search space for the user selection stage decreases from $(\sum_{k=1}^{K} N_k)$ as in [14] to K, when our proposed algorithm is employed.

VII. Conclusion

In this paper, we have considered the co-existence between cognitive users and a primary user. We have proposed an opportunistic spectrum sharing approach in order to maximize the sum-rate throughput of the cognitive system and minimize the interference to the primary user. Particularly, the proposed approach is based on a user selection algorithm, along with a ZFB technique and a water-filling power allocation. The proposed user selection algorithm schedules the users that are δ_p-orthogonal to the primary user and δ_c-orthogonal to each other. Numerical results have shown that the proposed method is capable of achieving a high sum-rate throughput while protecting the primary user, with low complexity. Furthermore, the MIMO case has been investigated where each cognitive user is equipped with multiple antennas. In such scenario, simulation results have shown that our proposed method combined with receive antenna selection is capable of further reducing the complexity of the selection with little loss in the sum-rate throughput.

The results of the paper were based on the assumption of perfect channel state information at the transmitter (CBS), which may not be a practical assumption. Therefore, an interesting topic remains open for future consideration, which is the investigation of the robustness of the proposed broadcast scheduling algorithm with respect to channel estimation errors. Besides, since the optimal solution to our formulated problem can not be provided, we could only propose a heuristic algorithm, that can give a sub-optimal solution. The proposed algorithm is able to select the users that maximize the cognitive system throughput, while making a little interference to the primary user, depending on the parameters δ_p and δ_c.

Appendix I

Let $\tilde{\mathbf{h}}_p = \frac{\mathbf{h}_p}{||\mathbf{h}_p||}$. We can decompose $\mathbf{h}_k$ into $\tilde{\mathbf{h}}_p$ and its orthogonal components $\tilde{\mathbf{h}}_p^{\perp}$ as follows $\mathbf{h}_k = \mathbf{h}_k^{||}\tilde{\mathbf{h}}_p + \tilde{\mathbf{h}}_p^{\perp}\mathbf{h}_k^{\perp}$, where $|\mathbf{h}_k^{||}|^2 \backsim \Gamma(1,1)$ and $||\mathbf{h}_k^{\perp}||^2 \backsim \Gamma(M-1,1)$. $\mathbf{x} \backsim \Gamma(p,\lambda)$ means that $\mathbf{x}$ is distributed according to the gamma distribution with parameters (p,λ). Furthermore, $|\mathbf{h}_k^{||}|^2$ and $||\mathbf{h}_k^{\perp}||^2$ are independent. It is easily shown that $\mathbf{h}_k^{||} = \tilde{\mathbf{h}}_p^{\mathcal{H}}\mathbf{h}_k$ and $\mathbf{h}_k^{\perp} = (\tilde{\mathbf{h}}_p^{\perp})^{\mathcal{H}}\mathbf{h}_k$. Since $||\mathbf{h}_k||^2 = |\mathbf{h}_k^{||}|^2 + ||\mathbf{h}_k^{\perp}||^2$, we can write

$$|\Delta(\mathbf{h}_p,\mathbf{h}_k)|^2 = \frac{|\mathbf{h}_p^{\mathcal{H}}\mathbf{h}_k|^2}{||\mathbf{h}_p||^2||\mathbf{h}_k||^2} = \frac{||\mathbf{h}_k^{||}||^2}{||\mathbf{h}_k^{||}||^2 + ||\mathbf{h}_k^{\perp}||^2}.$$

It is known that, if Y_1 and Y_2 are independently distributed with $\Gamma(a,\lambda)$ and $\Gamma(b,\lambda)$, respectively, then, $Z = Y_1/(Y_1 + Y_2)$ has $\beta(a,b)$, where $\beta(a,b)$ is beta distribution [22]. By applying this, we can obtain

$$\frac{|\mathbf{h}_p^{\mathcal{H}}\mathbf{h}_k|^2}{||\mathbf{h}_p||^2||\mathbf{h}_k||^2} \backsim \beta(1, M-1).$$

Using the property that the CDF of $\beta(a,b)$ is the regularized incomplete beta function $I_x(a,b)$ [22], we obtain

$$\text{Prob}\{k \in \mathcal{Q}_1\} = \text{Prob}\left\{\frac{|\mathbf{h}_p^{\mathcal{H}}\mathbf{h}_k|^2}{||\mathbf{h}_p||^2||\mathbf{h}_k||^2} \le \delta_p^2\right\} = I_{\delta_p^2}(1, M-1).$$

From the definition of the regularized incomplete beta function [22], we can get

$$I_{\delta_p^2}(1, M-1) = \frac{B(\delta_p^2, 1, M-1)}{B(1, M-1)},$$

where $B(x,a,b)$ and $B(a,b)$ are the incomplete beta function and the beta function, respectively. Specifically, we have

$$B(x,a,b) = \int_0^x t^{a-1}(1-t)^{b-1}dt, \text{ and}$$
$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt.$$

Therefore, we obtain $\text{Prob}\{k \in \mathcal{Q}_1\} = 1 - (1-\delta_p^2)^{M-1}$.

APPENDIX II

Define $\mathcal{L}_{c_i} = \{\mathbf{h}_j | j \in \mathcal{S}\}$ and $\tilde{\mathcal{L}}_{c_i} = \left\{\frac{\mathbf{h}_j}{||\mathbf{h}_j||}, \forall \mathbf{h}_j \in \mathcal{L}_{c_i}\right\}$. We can decompose $\mathbf{h}_k$ into $\tilde{\mathcal{L}}_{c_i}$ and its orthogonal components $\tilde{\mathcal{L}}_{c_i}^{\perp}$ as follows

$$\mathbf{h}_k = \tilde{\mathcal{L}}_{c_i}\mathbf{h}_k^{||} + \tilde{\mathcal{L}}_{c_i}^{\perp}\mathbf{h}_k^{\perp},$$

where $||\mathbf{h}_k^{||}||^2 \backsim \Gamma(i,1)$ and $||\mathbf{h}_k^{\perp}||^2 \backsim \Gamma(M-i,1)$. Furthermore, $||\mathbf{h}_k^{||}||^2$ and $||\mathbf{h}_k^{\perp}||^2$ are independent. It can be shown that $\mathbf{h}_k^{||} = (\tilde{\mathcal{L}}_{c_i})^{\mathcal{H}}\mathbf{h}_k$ and $\mathbf{h}_k^{\perp} = (\tilde{\mathcal{L}}_{c_i}^{\perp})^{\mathcal{H}}\mathbf{h}_k$. Since $||\mathbf{h}_k||^2 = ||\mathbf{h}_k^{||}||^2 + ||\mathbf{h}_k^{\perp}||^2$, we can write

$$\frac{||(\mathcal{L}_{c_i})^{\mathcal{H}}\mathbf{h}_k||^2}{||\mathcal{L}_{c_i}||^2||\mathbf{h}_k||^2} = \frac{||\mathbf{h}_k^{||}||^2}{||\mathbf{h}_k^{||}||^2 + ||\mathbf{h}_k^{\perp}||^2}.$$

Because each channel in $\mathcal{L}_{c_i}$ satisfies the δ_c-orthogonality, we can get the following

$$\text{Prob}\{k \in \mathcal{Q}_i\} = I_{\delta_c^2}(i, M-i).$$

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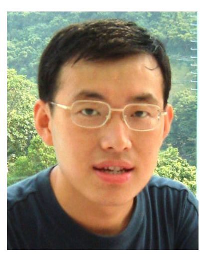

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